Year 1 – Week 20 Exam Questions

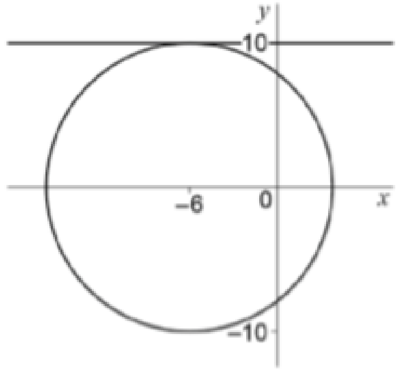
Mark Scheme

2	Circles correct answer	AO1.1b	B1	12
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3 (a)	Selects a correct method – differentiates to obtain $\frac{dy}{dx}$ Or completes the square or uses the calculator to locate the minimum point	AO3.1a	M1	$\frac{dy}{dx} = 4x + 5$ At minimum point, $4x + 5 = 0$ $x = -\frac{5}{4}$ $2\left(-\frac{5}{4}\right)^2 + 5\left(-\frac{5}{4}\right) + k = -\frac{3}{4}$ $k = \frac{19}{8}$ **Alternative solution** $y = 2\left[(x + \frac{5}{4})^2 - \frac{25}{16}\right] + k$ $y = 2(x + \frac{5}{4})^2 - \frac{25}{8} + k$ Using stated minimum value $-\frac{25}{8} + k = -\frac{3}{4}$ $k = \frac{19}{8}$
	Obtains the correct derivative and sets the expression equal to 0 to solve for x Or completes the square correctly/uses the calculator to find the x-coordinate of the minimum point	AO1.1b	A1	
	Forms an appropriate equation to find k	AO1.1a	M1	
	Finds the correct value of k	AO1.1b	A1	
	Total		**4**	
3 (b)	States the correct value of d	AO1.1b	B1	$d = \frac{3}{4}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6	Rewrites into separate terms and differentiates the expression	AO3.1a	M1	$f(x) = x^{-5} - 2x^4$ $f'(x) = -5x^{-6} - 8x^3$
	Differentiates at least one term correctly	AO1.1a	M1	$f'(x) = \frac{-5}{x^6} - 8x^3$ $f'(x) = -\left(\frac{5}{x^6} + 8x^3\right)$
	Differentiates both terms correctly	AO1.1b	A1	For $x > 0$, $\frac{5}{x^6}$ is always positive
	Uses a correct method and deduces why the expression for the derivative is always negative	AO2.2a	R1	$8x^3$ is always positive Hence $f'(x) < 0$, for $x > 0$ $f(x)$ is a decreasing function
	Clearly states the definition of a decreasing function within their explanation	AO2.4	E1	$\Leftrightarrow$ $f'(x) < 0$ for all values of x in that interval hence $f(x)$ is a decreasing function
	Writes a clear rigorous argument that links the steps together and makes a clear deduction about the gradient being negative for $x > 0$ and explains clearly that this satisfies the condition for the function to be decreasing	AO2.1	R1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7 (a)(i)	Begins to form a single equation by using substitution to eliminate y (or x)	AO3.1a	M1	By substitution $x^2+(mx+10)^2+12x=64$ $x^2+m^2x^2+20mx+100+12x-64=0$ $(m^2+1)x^2+(20m+12)x+36=0$ For distinct solutions $b^2-4ac>0$ hence $(20m+12)^2-4(36)(m^2+1)>0$ $(20m+12)^2-144(m^2+1)>0$
	Expands brackets and collects terms to form a quadratic equation	AO1.1b	A1	
	States that a condition for distinct solutions means that the quadratic has distinct real roots and so $b^2-4ac>0$	AO2.4	R1	
	Writes a clear rigorous argument that links the steps together and forms an inequality from the discriminant to obtain the printed result	AO2.1	R1	
	Total		**4**	
7 (a)(ii)	Selects an appropriate method to solve the inequality – by sketching, tabulating or use of a calculator	AO1.1a	M1	$(20m+12)^2-144(m^2+1)>0$ $400m^2+480m+144-144m^2-144>0$ $256m^2+480m>0$ $m(8m+15)>0$ $m>0, m<-\frac{15}{8}$
	Solves inequality correctly	AO1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7 (b)(i)	Completes the square to identify centre and radius	AO1.1a	M1	$(x+6)^2+y^2=100$ Centre $(-6, 0)$ Radius 10 
	Draws a circle on the diagram	AO1.1b	B1	
	Positions circle correctly on diagram with correct radius	AO1.1b	A1	
	Draws a horizontal line at $y = 10$	AO1.1b	B1	
	Total		**4**	
7 (b)(ii)	Explains that when $m = 0$ the line is a tangent to the circle	AO3.2a	B1	The line $y = 10$ touches the circle $x^2+y^2+12x=64$ and is therefore a tangent
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8 (a)(i)	Explains that $x-3$ is a factor implies that $\mathrm{f}(3)=0$ or $\mathrm{g}(3)=0$	AO2.4	E1	$x-3$ is a factor of $\mathrm{f}(x)$ hence $\mathrm{f}(3)=0$, giving $2(3)^3-11(3)^2+(p-15)(3)+q=0$ $54-99+3p-45+q=0$ $3p+q=90$
	Substitutes 3 for the value of x into both equations	AO1.1a	M1	
	Obtains two correct unsimplified equations	AO1.1b	A1	$x-3$ is a factor of $\mathrm{g}(x)$ hence $\mathrm{g}(3)=0$, giving $2(3)^3-17(3)^2+p(3)+2q=0$ $54-153+3p+2q=0$ $3p+2q=99$
	Completes the proof by simplifying the equations obtained with all notation being correct and no slips throughout	AO2.1	R1	
	Total		**4**	
8 (a)(ii)	Solves the given simultaneous equations correctly to find the values of p and q	AO1.1b	B1	$q=9$ $p=27$
	Total		**1**	
8 (b)	Adds the two functions together to obtain $\mathrm{h}(x)$	AO1.1b	B1	$\mathrm{f}(x)+\mathrm{g}(x)=4x^3-28x^2+39x+27$ $(x-3)$ is a common factor and by inspection $\mathrm{f}(x)+\mathrm{g}(x)=(x-3)(4x^2-16x-9)$ $=(x-3)(2x+1)(2x-9)$
	Extracts $(x-3)$ as a factor and attempts to find the corresponding quadratic factor	AO1.1a	M1	
	Obtains the correct quadratic factor	AO1.1b	A1	
	Factorises the expression for $\mathrm{h}(x)$ fully NMS – 4 marks	AO1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9 (a)	Identifies that the first error occurred when the $\cos^2\theta$ term was cancelled	AO2.3	R1	He cancelled the $\cos^2\theta$ term so did not consider solutions where $\cos^2\theta = 0$ which would have given two more solutions He took the square root of both sides of the equation and only considered the positive root so did not consider the possibility of a negative root, which would have given more solutions
	Explains that this could be equal to zero and would give more solutions	AO2.4	E1	
	Identifies that the second error occurred when the square root was taken of each side	AO2.3	R1	
	Explains that when taking the square root both positive and negative values should be considered and so Martin has missed the values that give a negative answer	AO2.4	E1	
	Total		**4**	

9 (b)	Selects a method to find more solutions of the equation	AO1.1a	M1	Either $\cos\theta = 0$ or $\sin\theta = -\frac{1}{2}$ When $\cos\theta = 0$ $\theta = 90°, 270°$ When $\sin\theta = -\frac{1}{2}$ $\theta = 210°, 330°$
	Solves $\cos\theta = 0$ correctly to obtain two more solutions	AO1.1b	A1	
	Solves $\sin\theta = -\frac{1}{2}$ correctly to obtain two more solutions	AO1.1b	A1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10	Circles correct response	AO1.1b	B1	40 kg
	Total		**1**	

11	Circles correct response	AO1.1b	B1	$\frac{2400}{6 \times 60} = 6.67\text{ m s}^{-1}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13 (a)	Draws a correct velocity time graph with correct shape	AO3.3	B1	
	Uses correct values and labels	AO1.1b	B1	
	Total		**2**	
13 (b)	Forms an equation to find time	AO3.4	M1	$11 = 8 + 2t$ $t = 1.5$
	Obtains correct total time	AO1.1b	A1	8 + 1.5 = 9.5 seconds
	Total		**2**	
13 (c)(i)	Suggests taking into account the length of the train	AO3.5c	B1	The length of the train could be included
	Total		**1**	
13 (c)(ii)	States that this will reduce the time found	AO3.5a	B1F	This would reduce the time found in **(b)** because the train would not have to travel so far.
	Total		**1**	

15 (a)	Forms equation of motion – condone sign errors	AO1.1a	M1	$204 - 19 \times 9.8 = 19a$ $a = 0.94 \text{ m s}^{-2}$
	Forms a fully correct equation	AO1.1b	A1	
	Finds the correct acceleration to two significant figures	AO3.2a	A1	
	Total		**3**	
15 (b)	Makes a correct deduction about the acceleration	AO2.2a	B1F	$a < 0.94 \text{ m s}^{-2}$
	Total		**1**	